

ON THE RESTRICTED EIGENVALUES CONDITION FOR GAUSSIAN MATRICES (FOR THE SPECIAL SESSION ON MATHEMATICAL STATISTICS)

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Résumé. Dans cet exposé, nous présenterons comment la condition RE (des valeurs propres restreintes) d'une matrice gaussienne centrée est reliée à celle de sa matrice de covariance.

Mots-clés. Lasso, condition RE.

Abstract. In this talk, we show the connection between the RE condition for a random Gaussian matrix with zero mean and the corresponding condition for its covariance matrix.

Keywords. Lasso, RE Condition.

1 High dimensional regression and Lasso

The Lasso is a popular method for estimation, prediction and variable selection in high-dimensional statistical problems. Originally, it has been introduced for solving the multiple linear regression problem with a large number of covariates. This corresponds to observing n iid pairs $(X_i, Y_i) \in \mathbb{R}^p \times \mathbb{R}$, with large p , and to aiming at finding a sparse vector $\beta^* \in \mathbb{R}^p$ such that

$$\mathbf{E}(Y_i | X_i = x) \approx \langle x, \beta^* \rangle.$$

The Lasso suggests to estimate β^* by solving the following convex optimization problem

$$\hat{\beta}^{\text{Lasso}} \in \arg \min_{\beta \in \mathbb{R}^p} \left(\frac{\|Y - X\beta\|_2^2}{2n} + \lambda \|\beta\|_1 \right),$$

where $Y = (Y_1, \dots, Y_n)$, $X = (X_1, \dots, X_n) \in \mathbb{R}^{n \times p}$ and $\lambda > 0$ is a tuning parameter.

2 Gaussian design and RE

It has been proved by Bickel *et al.* (2009) that the Lasso achieves the fast rate of convergence if the design matrix X satisfies the restricted eigenvalues (RE) condition.

When the design is random, imposing the RE condition to X is not suitable. Fortunately, if the rows of X are iid Gaussian vectors with zero mean and a covariance matrix Σ satisfying the RE condition, then one can show that with large probability the RE condition is satisfied for the random matrix X as well (see Rakutti *et al.* (2010)). The proof of the last claim is very elegant and makes use of a symmetrization argument that we will try to review in this talk.

Bibliographie

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