Parameter Estimation for Mixed-type Distributions with Application to Destruction Rate Modeling in Insurance

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Résumé. Dans le jargon actuariel, une courbe d'exposition est définie comme le ratio de l'espérance de perte limitée à différentes limites et de l'espérance de perte totale, i.e. $d \mapsto E(X \wedge d)/E(X)$. Nous présentons des modèles de taux de destructions, soit définis par leur fonction de répartition soit de manière équivalente définis par la courbe d'exposition. Une attention toute particulière est porté aux lois de probabilité "gonflées" en 1 et à la loi MBBEFD. L'estimation des paramètres de ces modèles est réalisée par la méthode de maximum de vraisemblance et la méthode des moments. Les propriétés des estimateurs sont ensuite étudiés. Enfin des illustrations numériques sont proposés dans le contexte actuariel.

Mots-clés. Taux de destruction, MBBEFD, lois mixtes, estimation ponctuelle

Abstract. Within actuarial jargon, an exposure curve is a distribution of the ratio between the limited expected loss at various limits and the unlimited expected loss, i.e. $d \mapsto E(X \wedge d)/E(X)$. We present destruction rate models either defined by their distribution function or equivalently by their exposure curve. A particular attention is given to one-inflated distributions and the so-called MBBEFD distribution. Parameter estimation for these two models is carried out by maximum likelihood estimation and moment matching estimation. Properties of these estimators are studied. Finally, numerical illustrations are given in the actuarial context.

Keywords. Destruction rate, MBBEFD, mixed-type distribution, point estimation.

1 Destruction rate models

We seek to model destruction rates of private properties (building, car, boat,...) linked to insurance losses (fire, floods,...). Within actuarial jargon, it is relevant to use exposure curves defined as the following ratio function $d \mapsto E(X \wedge d)/E(X)$ for a random variable X. Typically, they are used to rate exposures from large commercial risks and in non-proportional reinsurance treaties for non-life business, see e.g. [2, 3, 4].

It is rather common to assume that X lies in the unit interval so that X represents the percentage of loss with respect to the maximum insured loss amount. Therefore, attention is given to finite-support distribution such as the beta and the shifted truncated Pareto distributions.

Then, we present some examples of exposure curves for classic continuous probability distributions. We also emphasize some pitfalls of using such distributions since observations generally have mass of probability at some values, e.g. $P(X = 1) > 0$. Using full discrete distributions does not solve the problem since there are no particular probability masses for values $X \in]0, 1[$.

In order to deal with these specificities, we introduce the exposure curve, particularly appropriate for mixed-type distributions. Then we compute exposure curves for one-inflated distributions of the previously introduced distributions, such as the one-inflated beta distribution.

Finally, we present a parametric distribution called MBBEFD introduced by [1] defined in terms of the exposure curve. The acronym MBBEFD stands for Maxwell Boltzmann Bose Einstein Fermie Dirac and refers to distributions from statistical physics. Two parametrizations of the MBBEFD distribution are in fact studied.

2 Point estimation and application

After introducing possible distributions to model destruction rates, we present estimation procedures. We focus on classical methods (maximum likelihood estimation, moment or quantile matching estimations) and compare those methods.

We first investigate parameter estimation of one-inflated distributions in a general setting, and then deal with special cases such as the one-inflated beta distribution. Secondly, we turn our attention to the estimation of parameters for the MBBEFD distribution and its two parametrizations.

We consider a simulation study in order to assess bias and variance of the estimators for each distribution considered as well as to benchmark theoretical properties.

Finally, we illustrate these estimators on a real dataset of destruction rates of property insurance. This example show the advantages and drawbacks for each distribution. All numerical illustration are carried out in R with an R package [5] under development on github.

Bibliography

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